

A Study on Evaluation Metrics for Multi Criteria Decision Making (MCDM) Methods - TOPSIS, COPRAS & GRA

A. Martin¹, T. Miranda Lakshmi², V. Prasanna Venkatesan³

¹Assistant Prof., Dept. of Computer Science, Central University of Tamil Nadu, Thiruvavur

²Research Scholar, Research and Development Centre, Bharathiyar University, Coimbatore

³Professor, Department of Banking Technology, Pondicherry University, Puducherry

Abstract - Metrics are units of measurement. It is frequently used to mean a set of specific measurements taken on a particular process. They are very important to estimate the performance of any application. In this study, Multi Criteria Decision Making (MCDM) methods such as Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Complex Proportional Assessment (COPRAS) and Grey Relational Analysis (GRA) are taken into consideration. MCDM methods are applied to solve decision problems with different number of conflicting criteria. Generally these techniques are evaluated using the parameters such as time complexity, space complexity, sensitivity analysis and rank reversal. In addition to these existing evaluation parameters two new evaluation parameters such as rank occurrence and repeated ranking are designed. Hence metrics are designed for these evaluation parameters. To apply these metrics on selected MCDM methods, a case study is conducted on teachers evaluation based on the criteria such as subject knowledge, interaction with students, time management, communication skills, pedagogy of teaching, care and attention and dedication and involvement. These metrics are applied successfully in these selected methods and TOPSIS attained better performance when it is compared to other methods.

Keywords — Multi Criteria Decision Making (MCDM), TOPSIS, COPRAS, GRA, Evaluation Metrics, Estimation, sensitivity analysis, rank reversal, teacher's evaluation

1. INTRODUCTION

MCDM techniques evaluate the ranking order of the alternatives based on the conflicting criteria [1]. It is one of the widely applied decision making technique under different conditions [2]. It also constructs a global preference relation for a set of alternatives with respect to different criteria [3]. MCDM uses different kinds of techniques to identify a best alternative. The widely applied techniques are AHP, ELECTRE, GRA, TOPSIS, PROMTHEE and so on. In order to apply these techniques to different applications, evaluation parameters are required. From the literature it has been found that there is no evaluation metrics are designed for MCDM techniques [8]. Hence in this research, popularly applied MCDM techniques such as TOPSIS, COPRAS and GRA are selected and evaluation metrics are designed. To better explain the design of metrics for selected techniques a case study on performance evaluation of faculties is considered. The rest of the paper is set out as follows. In section 2, the literature is reviewed on metrics of MCDM techniques, section 3 describes the proposed methodology, section 4 describes the experimental design, and section 5 presents the result and discussion and eventually, ends up with conclusion, findings of the study and the future research.

2. PRIOR RESEARCH

Numerous MCDM methods are developed to solve the real – world decision problems. From the survey very limited work on metrics has been found for TOPSIS, COPRAS and GRA [9]. This literature survey studies various evaluation parameters that have been designed for MCDM methods. The popularly applied evaluation parameters are sensitivity analysis, ranking reversal, time complexity and space complexity [10]. This research also studies the functionalities of the selected three MCDM methods to propose new metrics. The next section describes about the various process involved in TOPSIS, COPRAS and GRA.

2.1 Topsis

TOPSIS method is widely applied which finds better alternative across different application domains. Many applications are evaluated using TOPSIS-AHP and TOPSIS-Fuzzy AHP methods.

- It is possible to combine (hybrid) TOPSIS with other kinds of MCDM method. From the literature review very limited work has been found to simplify the TOPSIS [1].
- There are many normalization techniques, in which only vector normalization is used in TOPSIS and COPRAS methods [11].
- Sensitivity analysis is the parameter which is most widely used in evaluating the performance of the MCDM method [12].

In order to simplify the generalized TOPSIS, it various processes are studied. On simplifying TOPSIS process it ranks the alternatives efficiently.

2.2 Copras

The COPRAS method is implemented in many applications. This technique has been applied for the selection of wind farm based on COPRAS-F method [13]. Location selection for wind farm takes an important role on power generation using wind energy [14]. However, the location selection is a complex multi- criteria problem due to the criteria factors which are conflicting in nature as well as uncertain. The process becomes more complex when a group of decision makers are involved in decision making. COPRAS is one of the well-known MCDM method, which identifies the better alternative directly and proportionally by determining a solution from the best solution and ideal least solution.

In this method, criteria weights and the alternatives ratings are taken into account as crisp numerical data [15]. It is difficult to get crisp data for different kinds of systems. These crisp numerical data cannot be applied to decision problems where the decision making involves with fuzziness (uncertainty found in problem space). These make the decision-making problems erroneous and inaccurate.

2.3 GRA

Grey Relational Analysis (GRA) model is proposed in order to prioritize quality attributes that are expected to influence performance level of a teacher [11]. GRA has been implemented for supply chain management which integrates suppliers, manufactures and distribution centres to get the suitable product to the suitable place at the expected time in the right condition [12]. The different criteria considered are supply chain risk, reduce production costs, make the maximum revenue, improve customer service, optimize inventory level, and improve business process which ends in increasing competitiveness, customer satisfaction and profitability [13].

3. PROPOSED WORK

One of special characteristics of TOPSIS is that it identifies the best alternative using distance metric. It considers two kinds of distances such as positive ideal solution (PIS) and negative ideal solution (NIS). The alternatives are ranked based cost criteria and benefit criteria. PIS decrease the cost criteria whereas NIS increases the cost criteria. Another important factor about PIS and NIS are benefit criteria. The benefit criteria increases when PIS is applied and it decreases when NIS is applied. From PIS and NIS Relative Closeness Coefficient (RCC) is computed which is used to rank the alternatives.

The core process of COPRAS method is finding the minimization index and maximization index [15]. Here, ranking is evaluated based on utility degree. This method has several advantages which includes less computational time, simple and transparent, high possibility of graphical interpretation, etc. GRA uses a specific concept of information [16]. It defines situations with no information as black, and those with perfect information as white. However, neither of these idealized situations ever occurs in world problems[8]-[10].In order to evaluate the faculty performance, the parameters such as time complexity, space complexity, sensitivity analysis and ranking reversal are considered. The main propose is to rank the alternatives based on criteria such as subject knowledge, interaction with students, time management, communication skills, pedagogy of teaching, care & attention and dedication & involvement.

4. EXPERIMENTAL DESIGN

Evaluating teaching performance is a key way to improve teaching quality and can play a vital role in the development of education institutions. A case study is conducted to illustrate the research proposal. Data has been collected from different students about the faculty performance based on different criteria. The results of proposed application results more scientific, accurate, and objective. It is expected that this work may serve as a supporting tool for administrators of higher education institutions in improving the educational quality level.

This research presents the results of a study that diagnoses and evaluates the performance of faculty from the perspective of Computer Science students of St. Joseph's College of arts and Science, Cuddalore. The research data is characterized by data collection procedures. The analysis and the data interpretation are quantitative. The research involved a population comprised of more than 1000 amongst which 140 students of computer science department are taken for illustration. The evaluation was performed based on the criteria such as subject knowledge, interaction with students, time management, communication skills, and pedagogy of teaching, care & attention and dedication and involvement.

4.1 Alternatives

The staff members of Computer science department are considered as alternatives. They are evaluated by the students' feedback opinion. The different criteria considered in this research are described as follows.

4.2 Criteria

Subject Knowledge: describes the in depth knowledge of a faculty on the subject which includes updating and growing advancements in contemporary topics related to the subject.

Interaction with students: evaluates the interpersonal relationship of the faculties with all the students. It includes easy approachability, amiability and interaction.

Time management: analyses how effectively a faculty manages time including regularity, and punctuality.

Communication skill: valuates how effectively and efficiently a faculty communicates with each other.

Pedagogy of teaching: describes the methods one uses to make teaching learning process interesting, interactive and understanding.

Care and Attention: shows whether the faculty is caring and attentive towards all the students without being partial towards a few.

Dedication & Involvement: measures how much dedicated, sincere and involved a faculty is towards work and others.

These evaluation parameters are applied to evaluate the faculty performance. All the selected three techniques are applied to find the most preferred faculty. These three techniques are evaluated using metrics such as time complexity, space complexity, sensitivity analysis and ranking reversal. It has been applied for faculty assessment to identify the most preferable faculty who satisfies all criteria in classroom teaching. In this research, the selected three methods are implemented with faculty performance data and from the better performing faculty is identified. The results are discussed in the next section.

4. RESULTS AND DISCUSSION

The ranking of alternatives is obtained using selected three methods with MCDM evaluation metrics. The results obtained for RCC with respect to PIS and NIS are described as follows.

Positive ideal solution:

0.0589 0.02506 0.03019 0.02968 0.02960 0.03125 0.02887

Negative ideal solution:

0.02947 0.00501 0.00604 0.00594 0.00592 0.00625 0.01155

Distance from Positive Ideal Solution

Columns 1 through 9

0.02274 0.02627 0.02690 0.03616 0.05509 0.02434 0.02357 0.02357 0.02357

Columns 10 through 13

0.02357 0.01860 0.02199 0.02624

Distance from Negative Ideal Solution

Columns 1 through 9

0.04131 0.04576 0.04390 0.02540 0.02357 0.05272 0.05509 0.05509 0.05509

Columns 10 through 13

0.04285 0.05388 0.04328 0.04662

Relative Closeness Coefficient

Columns 1 through 9

0.6450 0.6353 0.6201 0.4126 0.2997 0.6842 0.7003 0.7003 0.7003

Columns 10 through 13

0.6451 0.7434 0.6631 0.6399

The ranking of alternatives are as follows:

A11->A7->A8->A9->A6->A12->A10->A1->A13 ->A2->A3->A4->A5

Analysis of COPRAS Method:

The method COPRAS is analysed and the results are generated.

Utility Degree of Each Alternative n =

80.0000 88.6318 85.8400 54.1506 22.7918 97.0448 100.0000 100.0000 100.0000 80.4945 94.3234
 82.7918 88.7694

The following are the ranks of alternatives:

A7->A8->A9->A6->A11->A13->A2->A3->A12 ->A10->A1->A4->A5

Analysis of GRA Method

The GRA method is applied to estimate the weight of each criterion. This method will definitely improve the efficiency of the algorithm.

Grey Relational Grade

degree = Columns 1 through 9

0.59738 0.60103 0.60544 0.57630 0.55160 0.61038 0.61500 0.61500 0.61500

Columns 10 through 13 - 0.59942 0.61597 0.59297 0.60881

Ranking

The ranking of GRA method are as follows:

A7->A8->A9->A11->A6->A13->A3->A2->A10 ->A1->A12->A4->A5

The time and space complexity obtained for evaluation of faculty performance using three methods are described in Table 1.

Table 1.: Space and Time complexity obtained for the selected three methods.

Evaluation Parameters	MCDM Methodology		
	TOPSIS	COPRAS	GRA
Time Complexity	0.014925	0.044374	0.059576
Space Complexity	2536	3152	3360

From Table 1, it has been found that TOPSIS is taken very less time and space to identify the faculty performance. Next to TOPSIS, COPRAS has taken less space and time to find the better performing faculty. The results obtained for sensitivity analysis is described as follows.

4.1 Sensitivity Analysis

The sensitivity analysis is conducted in different ways by altering the weights. Generally, the total sum of the weight is always equal to one. The Sensitivity analysis is conducted in TOPSIS, COPRAS and GRA method to check robustness of the solution obtained using these methods. The result of sensitivity analysis is shown in Table 2, when all the weights have changed to 1. The results obtained for TOPSIS is described in Figure 1.

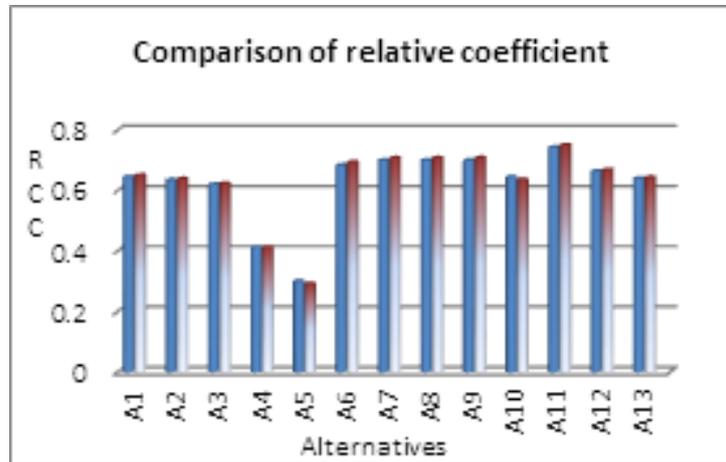


Figure 1: Sensitivity Analysis in TOPSIS

The rank for the alternatives has been changed in TOPSIS when conducting Sensitivity analysis. Now, the ranks are A11-A7-A8-A9-A6-A12-A1-A13-A2-A10-A3-A4-A5.

COPRAS

In the same way, Sensitivity Analysis is conducted in COPRAS method; the rank of following alternatives is changed. The rank of alternatives is described as follows A7-A8-A9-A6-A11-A13-A2-A3-A12-A1-A10-A4-A5 which is described in Figure 2.

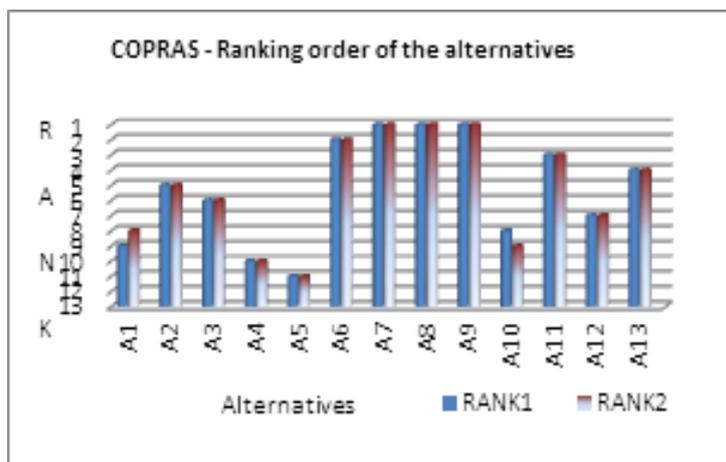


Figure 2: Sensitivity Analysis in COPRAS Method for evaluate the faculty performance

GRA

Finally, Sensitivity Analysis is carried out in GRA method, wherein the ranks of the alternatives are changed as follows: A7-A8-A9-A6-A11-A13-A2-A3-A12-A10-A1-A4-A5 which is described in Figure 3.

The ranking order changes in the alternatives is described in Figure 1, Figure 2 and Figure 3. In this analysis, it has been found that very minimum level of changes observed in the ranking order of the alternatives when it is

compared to COPRAS and GRA. The evaluation parameter “ranking reversal” is applied to faculty performance evaluation and results are described.

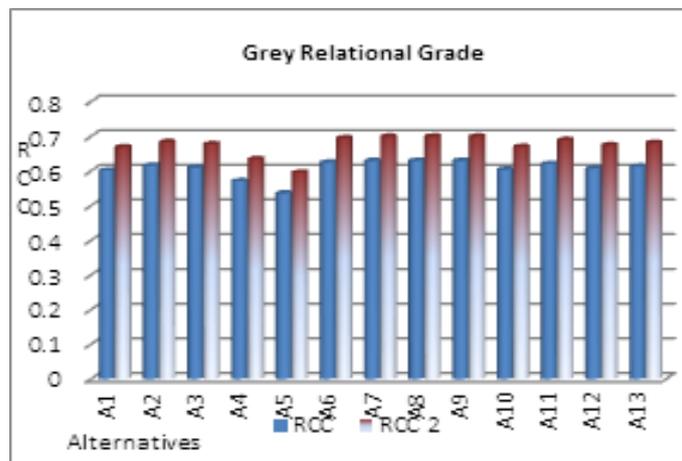


Figure 2 Sensitivity Analysis in GRA

4.2 Rank Reversal

Rank reversal is property of adding and removing the alternatives are taken for evaluation of faculty performance. It is applied in TOPSIS, COPRAS and GRA methods to validate the changes in the ranking order of alternatives.

Adding an Alternative - An identical copy of a best alternative is introduced to check and see if the indication of the best alternative changes or not.

Removing an Alternative- A worst alternative is removed in order to check the ranking getting change or not. To perform this, the worst alternative is found and removed from the original data.

TOPSIS – Adding an Alternative

The changing order of alternatives while adding an alternative using TOPSIS is described in Table 2.

Table2 Ranking Reversal in TOPSIS

Alter natives	*RCC	Ranks	RCC	Ranks in RR
A1	0.645	6	0.6501	5
A2	0.6353	8	0.6391	7
A3	0.6201	9	0.6237	9
A4	0.4126	11	0.4125	10
A5	0.2997	12	0.2918	11
A6	0.6842	3	0.6943	3
A7	0.7003	2	0.7082	2
A8	0.7003	2	0.7082	2
A9	0.7003	2	0.7082	2
A10	0.6451	5	0.6362	8
A11	0.7434	1	0.7499	1
A12	0.6631	4	0.6682	4
A13	0.6399	7	0.6434	6
A14	0.750	-	0.750	1

*RCC - Relative Closeness Coefficient

TOPSIS - Removing an Alternative

A worst alternative A5 is removed in order to check the ranking getting change or not.

- To perform this, the worst alternative A5 is found and removed from the original data.
- The ranking of alternatives are given in the Table 3 after removing the worst alternative from the original data.

GRA (Adding and removing an alternative)

Table 3: Ranking reversal in GRA (Adding an Alternative)

Alternatives	Grey Relational Grade	Ranking	GRG Rank Reversal	Ranking
A1	0.6023	9	0.669	10
A2	0.616	4	0.684	5
A3	0.6104	6	0.677	7
A4	0.5713	10	0.635	11
A5	0.5366	11	0.596	12
A6	0.6254	2	0.695	3
A7	0.63	1	0.700	2
A8	0.63	1	0.700	2
A9	0.63	1	0.700	2
A10	0.6044	8	0.670	9
A11	0.6209	3	0.690	4
A12	0.608	7	0.675	8
A13	0.6138	5	0.681	6
A14	0.631	1	0.701	1

Table 4: Ranking reversal in GRA (Removing an Alternative)

Alternatives	Relational grade	Ranking	Rank reversal	Ranking
A1	0.6023	9	0.669	10
A2	0.616	4	0.684	5
A3	0.6104	6	0.677	7
A4	0.5713	10	0.635	11
A5	0.5366	11	0.596	-
A6	0.6254	2	0.695	3
A7	0.63	1	0.700	2
A8	0.63	1	0.700	2
A9	0.63	1	0.700	2
A10	0.6044	8	0.670	9
A11	0.6209	3	0.690	4
A12	0.608	7	0.675	8
A13	0.6138	5	0.681	6
A14	0.631	1	0.701	1

Rank Occurrence and Repeated Ranking

The results of rank occurrence and repeated ranking for the selected methods is described in Table 5. Topsis shows efficient ranking compared to other methods and the obtained rank occurrence metric value is 3.2978 and repeated ranking is 5.2510. Evaluation results of metrics for TOPSIS; COPRAS & GRA for faculty performance is described in Table 4.

Metric	Selected MCDM Methods		
	TOPSIS	COPRAS	GRA
Ranking occurrence	3.2978	5.8743	6.7439
Repeated Ranking	5.2510	8.4232	10.879

Table 5: Cumulative Results of Metrics for TOPSIS, COPRAS &GRA

MCDM Metrics	TOPSIS	COPRAS	GRA
Time Complexity	0.0149	0.0443	0.0595
Space Complexity	2536	3152	3360
Sensitivity analysis	0.7499	99.99	0.70
Rank reversal	0.750	99.9	0.7
Repeated ranking	5.2510	8.4232	10.879
Rank occurrence	3.2978	5.8743	6.7439
RCC	0.7434	-	-
Utility Degree	-	100	-
Relative Grade	-	-	0.6300

The proposed metrics are experimented with TOPSIS, COPRAS and GRA. As a result of this study, the proposed metrics works more efficiently with the TOPSIS method when compared to the other two methods. It takes lesser time and space to complete its process and while performing sensitivity analysis and rank reversal it gives better performance.

6. CONCLUSION

Multi Criteria Decision Making is widely used for decision making problem where there are several factors in obtaining the best solution and different methods are used for solving complex problem. The problem is to find the best faculty who satisfies all the criteria in classroom teaching. Several algorithms are available in MCDM approach, where TOPSIS, COPRAS and GRA are the most preferred methods when compared to other methods. The approach, not only identify important criteria for teacher's evaluation but also finds out incomplete stuff associated with a teacher who needs improvement in certain criteria. The overall quality index proposed in this study can be used for quantitative assessment of teachers' performance. This index helps the administrators of education while taking strategic decisions like recruitment and promotion of faculty for overall growth of the institutes. However, the study can be extended further to a broad based methodology by considering more number of criteria for evaluation. The methodology can also be employed for comparison of quality of faculty in different educational settings. The efficiency of methods is measured in terms of Time Complexity and Space Complexity. The proposed method attains a better result with respect to RCC, time and space complexity. As a result of the study, the proposed methods are realistic and convenient in predicting the right faculty in the computer science department with respect to multiple conflicting criteria and by implementation, it has been found A11 as the best alternative, who satisfies all criteria and A5 as the worst alternative in Evaluation of Faculty performance. Hence it is proved that TOPSIS method is the best method when compared with other two methods.

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